Cartel and Oligopoly Pricing of Nonreplenishable Natural Resources

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I. INTRODUCTION

Few real world market structures correspond well to either of the textbook polar cases of absolute monopoly and perfect competition. Formal models of two intermediate structures have been thoroughly and usefully analyzed under static conditions.[1] The first model assumes that a single firm or a stable cartel controls a sizeable fraction of industry capacity and faces a competitive fringe of many small suppliers, each too tiny to have any noticable effect on market price. Fringe members thus take price as beyond their control and choose output to maximize profit. The dominant firm or cartel maximizes its profit subject to the constraint imposed by fringe supply behavior. The second model deals with noncooperative or Cournot/Nash oligopoly. A finite number of sellers is assumed, each large enough to have some control over price. Market equilibrium is defined as a situation in which no individual seller can increase its profits by changing only its own output, given the outputs of the other sellers.

This essay is concerned with the implications of these structures in markets for nonrenewable natural resources.[2] Following Hotelling (1931) and numerous subsequent authors, we assume that the total reserves of the resource in the hands of each producer cannot be increased and are reduced by production. Demand and cost conditions, including the relevant rate of interest, are constant over time. In such a world, producers must rationally consider price or output paths over time, so that both models outlined above become non-zero sum differential games. In what follows, we examine solutions to the games implied by various assumptions.

Our work relies heavily on that of Salant (1976), who seems to have done the only theoretical analysis of either imcomplete cartel or noncooperative oligopoly in nonreplenishable resource markets.[3] Section 2 presents and extends his model of cartel equilibrium. Following Salant, it is assumed that the cartel and the competitive fringe have and use perfect information about prices, outputs, and reserves of all producers at each instant. Under this assumption, it is clear that all expectations must be realized.

The assumption of perfect information seems a bit unrealistic in this context, however, At the very least, it is difficult for suppliers to verify rivals' assertions about levels of reserves. Accordingly, Sections 3 considers the implications of relaxing the assumption that the competitive fringe knows the cartel's reserves. The cartel can lie about reserves, thus creating expectations about future prices and outputs that will later prove to have been incorrect. (The fringe suppliers are assumed to tell the truth, perhaps because they are all too small to lie profitably.) However we suppose that the fringe can monitor cartel sales. Thus, in order for a lie about reserves to be believed, the cartel must follow the output path that would be optimal for the announced reserve level. We derive the surprising result that the cartel's optimal policy is to tell the truth. Competitive monitoring of output (plus careful analysis of observed output decisions) is as effective a constraint on cartel behavior as is full information about reserves.

Section 4 presents some exploratory analysis of noncooperative oligopoly in natural resource markets, assuming perfect information. We find that if all sellers have equal reserves and equal extration costs, existence and uniqueness of equilibrium can be established and some comparative dynamic results obtained.

As it will become clear in what follows, various of our results depend on assumptions that are stronger than we would like. Moreover, many potentially important aspects of natural resource markets are missing entirely from our models: strategic or other changes in buyer behavior, exploration for new reserves, differences in cartel members' objectives, cartel stability problems, and others.[4] Much more work remains to be done on imperfect competition in nonrenewable natural resource markets.

2. THE BASIC CARTEL MODEL WITH FULL INFORMATION

2.1. Assumptions

Following Salant, (1976), a market for a nonreplenishable natural resource (such as oil or bauxite) is considered in which consumers purchase supplies at the same price from independent firms. Market transactions are described in terms of a Cournot-Nash, noncooperative game. Each supplier maximizes its own discounted profits, taking as given the sales path for all other firms. One group of firms which can presumably exercise some market power by virtue of its large and concentrated resource holdings forms a cartel which acts collusively to set prices or limit sales. The remainder of the world's resource stock is dispersed among a sufficiently large number of other suppliers so that a dominant firm model results. Small producers act like competitors who perceive prices as given in each time period and choose a sales path to maximize discounted profits. The cartel simultaneously chooses a price path, supported by cartel sales, to maximize discounted profits taking aggregate sales by the competitive fringe as given. An equilibrium sales or price path is said to exist whenever the cartel and each fringe supplier are simultaneously maximizing individual discounted profits while taking as given the sales path of all other firms.

Let $q(t)$ be total market sales, and $q^m(t)$ and $q^c(t)$ be the sales by the cartel and competitive fringe respectively at time t. The stationary inverse demand function for the resource $p(q)$, with $p'(q) < 0$, is characterized by whether elasticity, $\varepsilon(q)$, is decreasing, constant, or increasing as a function of q, as indicated below.

$$\text{(DE)}\quad \varepsilon'(q) < 0 \quad ; \quad \varepsilon(\bar{q}) = 1 \quad ; \; p(0) < \infty$$

$$\text{(CE)}\quad \varepsilon'(q) = 0 \quad ; \quad \varepsilon(q) > 1 \; (\forall q)$$

$$\text{(IE)}\quad \varepsilon'(q) > 0 \quad ; \quad \varepsilon(q) > 1 \; (\forall q)$$

Salant (1976) employs the (DE) specification which assumes a finite "choke price" p(0) and decreasing demand elasticity. The alternatives (CE) and (IE) are included here for comparision with (DE).[5]

Specifications (DE) and (CE) are fairly standard; (IE) corresponds to a demand schedule with elasticity increasing in consumption. A justification for this assumption is that for small quantities, demand may be inelastic if certain amounts of the resource are essential in the production of some goods. At lower prices, however, the resource may be used in other industries for which substitute inputs exist as well. Consequently the elasticity of aggregate demand may increase.

Letting, I, I^m, and I^c represent initial total market, cartel, and competitive fringe reserves, the basic model employs a full information assumption,

(I1) All suppliers can observe $\{q(t), q^m(t), q^c(t), I^m, I^c\}$.

Assumption (I1) is modified in section 3, to allow for partial observability of I^m by fringe extractors.

2.2 Charterization of Equilibria

Market equilibrium occurs when individual suppliers are each maximizing discounted profits taking the sales paths of all other extractors as given. This Nash equilibrium concept is particularly compelling if one assumes the existence of a future market for the resource, where extractors can contract for a stream of future shipments to maximize discounted profits subject to the announced shipments of other firms. For simplicity, zero marginal extraction costs are assumed. (Results presented here are basically unchanged with the introduction of constant positive marginal extraction costs. Details are omitted). Taking the sales paths of other firms as given, straightforward maximization of cartel and individual fringe supplier profits yield the necessary conditions (2.1)-(2.4) for an equilibrium.[6]

(2.1) $e^{-rt}MR^m(q^m,q^c) \equiv e^{-rt}\{p(q^m + q^c) + \frac{dp}{dq} q^m\} = MR_0^m$

(2.2) $\int_0^T q^m(t)dt = I^m$

(2.3) $e^{-rt}p(q^m + q^c) = p_0$ whenever $q^c > 0$

(2.4) $\int_0^S q^c(t)dt = I^c$

Given the competitive sales path, the cartel maximizes profits by equating discounted marginal revenue (from its excess demand curve) across all time periods as in equation (2.1). Equation (2.3) is an equilibrium arbitrage condition which must hold whenever price-taking competitive fringe suppliers operate in the market. Inventory constraints are given by equations (2.2) and (2.4) where T and S represent the extraction horizon for the cartel and competitive fringe, respectively.

The first thing to notice about this equilibrium is that $q^m(t) > 0$ whenever $q(t) > 0$. This stems from the fact that $MR^m(0, q^c) = p(q)$ so that q^m can never go from a positive to a zero value or from a zero to a positive value while $q(t) > 0$, without violating (2.1) and (2.3).

We need two more preliminary results. Let $\{\tilde{q}^m(t)\}$ $\{\tilde{q}^c(t)\}$ denote the paths that maximize discounted profits for the cartel and that would result in competitive equilibrium respectively, in the extreme cases where $I^m = I$ and $I^c = I$. The paths for the cartel and competitive fringe are then described by conditions (2.1) - (2.2) and (2.3) - (2.4) respectively (making the obvious adjustments that $q^c(t) = 0, (\forall t)$ if $I^m = I$ and so forth, where necessary). If in addition to (DE), (CE), and (IE), we assume that $MR = R'(q) > 0$, $R''(q) < 0$ where $R(q) = p(q)q$, we can establish

Lemma 1. Considering $\dot{\tilde{q}}^c(q)$ and $\dot{\tilde{q}}^m(q)$ as functions of q, we have $\dot{\tilde{q}}^c$, $\dot{\tilde{q}}^m < 0$ and

$$(2.5) \quad \dot{\tilde{q}}^c(q) \lesseqgtr q^m(q) \text{ as } \varepsilon'(q) \lesseqgtr 0,\ q > 0$$

$$(2.6) \quad \frac{\dot{MR}}{MR}(q) \lesseqgtr \frac{\dot{p}}{p}(q) \text{ as } \varepsilon'(q) \lesseqgtr 0,\ \ q = q^m(t)$$

Lemma 2.

$$(2.7)\ \tilde{q}^m(0) \gtreqless \tilde{q}^c(0) \text{ as } \varepsilon'(q) \gtreqless 0$$

where $MR = p + \frac{dp}{dp} q$. Proofs of these Lemmas follow readily by taking time derivatives of equations (2.1) and (2.3).

Using Lemmas 1 and 2, the Nash equilibrium sales paths for various demand specifications are as follows. Let $s(t) = I^m(t)/I(t)$ represent current cartel inventories as a fraction of total world inventories.

Case (DE). For $s(t) = 1$ or 0 we revert to the polar cases just discussed. Suppose $0 < s(0) < 1$. Whenever $q^c(t) = 0$, $MR^m = MR$ and (2.1) and (2.6) imply that $\dot{p}/p < r$. Consequently, fringe suppliers never delay the beginning of extraction since real prices are falling. From (2.1) and (2.3), $\dot{MR}^m/MR^m = \dot{p}/p$, or $MR^m = Kp$, $0 < K < 1$, whenever $q^m, q^c > 0$. Solving for q^c yields

$$(2.8) \qquad q^c = q(1-(1-K)\varepsilon(q)).$$

The existance of a finite choke price implies $\varepsilon(q) \to \infty$ as $q \to 0$. Consequently by equation (2.8) q^c(and $I^c(t)$) go to zero while q, q^m and I^m are still positive. Thus, initially both the competitive fringe and the cartel operate for a period during which prices rise at the rate of interest. Eventually, fringe supplies are exhausted, and the entire market is left to the cartel, which then operates as a monopoly with $\dot{p}/p < r$.

Case (CE). With $\varepsilon' = 0$, it is easy to verify from (2.8), (2.2) and (2.4) that $q^c(t) = (I^c(0)/I(0))q$, $q^m(t) = (I^m(0)/I(0))q$ and $\dot{MR}^m/MR^m = \dot{p}/p = r$ for all t. For given initial world reserves of the resource, $I(0)$, the equilibrium price and total sales path are independent of the distribution of resources between the cartel and fringe suppliers. The condition $\varepsilon' = 0$ implies $p(0) = \infty$, so that resources are allocated over infinite time at a constant real price.

Case (IE). Whenever $q^c(t) = 0$, (2.1) and (2.6) imply $\dot{p}/p > r$. Fringe suppliers delay extraction as much as possible, consistent with the eventual sale of all their inventories. Of course real (discounted) prices can only rise if speculators can't buy resources and store them for sale later on. This is probably a fair assumption in the case of oil. Once fringe extraction begins, equation (2.8) and the condition $\varepsilon'(q) > 0$ implies $q^c(t)$ is positive for $q > 0$. Since $\varepsilon'(q) > 0$ we have $p(0) = \infty$ so that resources are allocated over infinite time. In summary, if I^m is sufficiently large relative to I^c there may exist an initial period during which ($q^c = 0$, $q^m > 0$) and real prices rise.[7] Eventually the fringe suppliers enter the industry whereupon prices rise at a competitive rate.

Summarizing, in the instance of (DE), the market is initially characterized by competitive price movements, followed by a second period of monopoly control. The opposite sequence of events can occur in case IE. For cases DE and IE market equilibriums become more monopolistic in character as $s = I^m/I \to 1$, whereas the equilibrium price and total sales path are independent of s for the CE specification

2.3 Existence and Uniqueness of Equilibrium

This sub-section is devoted to establishing the following basic result; which generalizes the existence proof in Salant (1976):

Proposition 1: Given (DE), (CE), (IE) and (I1) a unique equilibrium exists characterized by (2.1) - (2.4).

Proof: Proof for the case (DE) is presented here, noting that the other cases are established by a similar argument.

(a) The inventory constraint for total reserves is written as

$$(2.9) \qquad \hat{I}(0,S) \equiv \int_0^S q(t)dt + \int_S^T q(t)dt = I$$

During the first interval, (0,S) resources are allocated competitively with q^m, $q^c > 0$ and $p/p = r$; during the second interval the allocation is monopolistic with $q^c > 0$ and $\dot{p}/p < r$.

Changing the variable of integration in (2.9) we obtain

$$(2.10) \qquad \hat{I}(q_0, q_1) = \int_{q_1}^{q_0} - [q/\dot{\tilde{q}}^c]dq + \int_0^{q_1} - [q/\dot{\tilde{q}}^m]dq = I(0)$$

where $\dot{\tilde{q}}^c$ and $\dot{\tilde{q}}^m$, calculated from equations (2.3) and (2.1) respectively, represent the competitive and cartel-dominated rates of change in sales, as a function of q, and $q(0) = q_o$, $q(S) = q_1$ and $q(T) = 0$.

(b) Rewriting the stock constraints for the case where all resources are either sold monopolistically or competitively, we obtain

(2.2') $$\int_0^{\tilde{q}^m(0)} [-q/\dot{\tilde{q}}^m]dq = I$$

(2.4') $$\int_0^{\tilde{q}^c(0)} [-q/\dot{\tilde{q}}^c]dq = I$$

Then from Lemmas 1 and 2, and equations (2.2') and (2.4')

(2.11) $$\hat{I}(q_0, q_1) > \int_{q_1}^{q_0} - [q/\dot{\tilde{q}}^c]dq + \int_0^{q_1} - [q/\dot{\tilde{q}}^c]dq \geq I(0)$$

whenever $q_0 \geq \tilde{q}^c(0)$

and

(2.12) $$\hat{I}(q_0, q_1) < \int_{q_1}^{q_0} - [q/\dot{\tilde{q}}^m]dq + \int_0^{q_1} - [-q/\dot{\tilde{q}}^m]dq \leq I(0)$$

whenever $q_0 \leq \tilde{q}^m(0)$

so that in order to satisfy (2.2') (2.4') and (2.10) we must have

(2.13') $$\tilde{q}^m(0) < q_o < \tilde{q}^c(0).$$

(c) From (2.10) we can represent q_1 as a function of q_0. Note that Lemmas 1 and 2 and equations (2.2') and (2.4') imply

(2.14) $$\hat{I}(q_0, q_0) > I > \hat{I}(q_0, 0)$$

Differentiating equation (2.10) with respect to q_1 yields

(2.15) $$\frac{d\hat{I}}{dq_1} = [q/\dot{\tilde{q}}^c - q/\dot{\tilde{q}}^m] > 0.$$

Together, (2.14) and (2.15) imply that for each q_0 there is a unique $q_1(q_0)$ determined by equation (2.10).

(d) From (c) we can show that there is a one-to-one relationship between I^c and q_0 for a given $I(0)$. Equations (2.1) and (2.3) imply $q^c(q_1) = 0$. From (2.8) we have

$$q^c = (1 - (1-K)\varepsilon(q))q,$$

which together with the $q^c(q_1) = 0$ condition implies

$$K = 1 - 1/\varepsilon(q_1(q_0)).$$

We can express I^c in terms of q_0 as

$$(2.16) \quad I^c(q_0) = \int_{q_1(q_0)}^{q_0} - (1 - (1-K)\varepsilon(q))q/\dot{\tilde{q}}^c \, dq$$

Note that $q_1(\tilde{q}^c(0)) = 0$ and $q_1(\tilde{q}^m(0)) = \tilde{q}^m(0)$ by equations (2.10), (2.2') and (2.4'). Then from equation (2.16) we obtain (assuming $s\varepsilon(0,1)$)

$$(2.17) \quad I^c(\tilde{q}^c(0)) = I(0) > 0 = I^c(\tilde{q}^m(0))$$

and differentiating (2.16) with respect to q_0 yields

$$(2.18) \quad \frac{dI_c}{dq_0} = \frac{q(1-(1-K)\varepsilon(q_0)}{\dot{q}(q_0)} + \int_{q_1}^{q_0} - \frac{\frac{dK}{dq_1}\frac{dq_1}{dq_0}}{\dot{q}} \varepsilon(q)\, dq > 0$$

Together (2.17) and (2.18) imply there exists a unique q_0 for each I^c and, conversely, for each q_0 there is a unique I^c.

In summary, given initial stocks (I, I^c) there exists a unique q_0 satisfying equation (2.16). But q_0 maps into a unique value for q_1 to satisfy equation (2.10). Given q_0 and q_1 the equilibrium is completely described by equation (2.1), (2.2'), (2.3), (2.4'), and the proof of Proposition 1 is complete.

2.4 Properties of Equilibrium

Denote p_0 and MR_0^m as the constant discounted price and marginal revenue accruing to fringe suppliers and the cartel respectively in market equilibrium and let $W^m(I^m, I^c)$ and $W^c(I^m, I^c)$ represent total discounted profits earned by the cartel and competitive fringe sector. Then the qualitative implications of the market equilibrium equations (2.1)-(2.4) for the comparative static effects of changes in initial inventory levels are presented in Table 1. Details of the sign derivations for the first five rows of Table 1 derived following the methods of Hartwick (1977), are presented in Appendix A for use later on.

Lines 1 and 2 of Table 1 indicate that cartel and competitive fringe marginal values of reserves, measured by MR_0^m and P_o, respectively, decline with greater initial inventories. Total profits for each sector vary directly (inversely) with own initial reserves (reserves of the other sector) as revealed in lines 3 and 4. The last three lines of Table 1 imply that total industry profits and individual profits for all suppliers increase as S, increases, whenever equilibrium prices are affected by the division of initial reserves between the two sectors. The larger the cartel is relative to market, the better off are all suppliers in aggregate.

Thus the formation of a cartel benefits all extractors including the fringe suppliers. Yet one can show that fringe suppliers can generally earn greater profits than cartel members, that individual members can profitably defect from the cartel, assuming others maintain the cartel discipline, and that therefore cartels are basically unstable here, as they are in static textbook models. The fringe supplier is able to sell his output at some constant real price p_0. Differentiating equations (2.1) and (2.3) with respect to time yields

$$\dot{p}/p \lesseqgtr r. \quad \text{as } \varepsilon' \lesseqgtr 0 \text{ for } q^c = 0,\ q^m > 0$$

$$\dot{p}/p = r \qquad \text{for } q^c,\ q^m > 0$$

implying that for cases DE (IE), the cartel must sell some of its stock during a final (initial) extraction phase at discounted values below p_0. Thus, the average present value of cartel reserves is less than that of reserves held by competitors. In the constant elasticity case, individual extractors receive the same discounted price, independent of the distribution of initial reserves.

TABLE 1

SIGNS OF COMPARATIVE STATIC DERIVATIVES

Endogenous Variables	Exogenous Variables: I^c	I^m	$S = I^m/I$ [a]	(0) [b]	[c]
MR_0^m	-	-	-	(0)	-
P_0	-	-	+	(0)	-
W^m	-	+	+	(0)	+
W^c	+	-	-	(0)	-
$W^m + W^c$	+	+	+	(0)	+
$\frac{W^m}{I^m}$	-	-	+	(0)	+
$\frac{W^c}{I^c}$	-	-	+	(0)	+

[a] Comparative static derivatives for (DE) case

[b] The zeroes in parenthesis indicate that comparative static derivatives are zero in those cases (CE) and sometimes (IE) where changes in s do not affect the price and total quantity paths in equilibrium.

[c] Comparative static derivatives for the (IE) case where changes in s affect price and total quantity paths in equilibrium. Results are derived assuming $d/dq(MR(q)) < 0$.

3. A CARTEL MODEL WITH PARTIAL OBSERVABILITY OF INITIAL INVENTORIES

3.1 Introduction

In the previous section, market strategies were examined for a game with complete information, in which each of the suppliers knew the sales and initial inventories of all the others. Because there is significant uncertainty about the world supply of some resources like oil, it seems appropriate to consider situations in which there is partial knowledge of inital inventories. In this section, we assume (a) initial competitive fringe supplies are known by all extractors, but cartel inventories are observed only by the cartel and (b) for strategic purposes, the cartel can misrepresent its initial inventories to fringe suppliers so long as the cartel's sales path is "consistent" with its announced inventory levels. The case considered here gives rise to a sophisticated-naive, leader-follower model, reminiscent of the static Stackelberg model. The cartel chooses a best inventory and sales strategy taking the response of the fringe suppliers into account. The justification for treating the two sectors differently is that individual fringe suppliers perceive that they can not alter prices by acting strategically, where as the cartel recognizes its ability to affect prices by announcing different inventory and sales paths. The model of cartel pricing under incomplete information analyzed in this section is characterized by the assumption,

(I2) Cartel suppliers can observe $q^c(t)$, I^c

Competitive Fringe suppliers can observe $q^m(t)$ and the

and the following market procedure:

(P1) At time t_0, the cartel announces a level of initial stock $\bar{I}^m$.[8] Assuming I^c is known, trade for current and future sales of the resource by both continues until the sales paths, $q^m(\bar{I}^m, I^c, t)$ and $q^c(\bar{I}^m, I^c, t)$, satisfying the conditions (2.1) to (2.4) for a market equilibrium are announced by the cartel and competitive fringe sectors. From Proposition 1 we know that these path exist and are unique for each pair $(\bar{I}^m, I^c)$. The market remains in equilibrium, and fringe extractors continue to sell equilibrium quantities $q^c(\bar{I}^m, I^c, t)$ based on the assumption that $\bar{I}^m$ is the true cartel inventory level, until the cartel deviates from its equilibrium path. At this point, the cartel is forced to reveal its true current inventory level, and the market comes to a new equilibrium based on actual reserve levels.

Given (P1), one cartel strategy is to announce an initial stock $\bar{I}^m$ which is larger than the actual cartel inventory. Once a market equilibrium is established based on initial stocks $(\bar{I}^m, I^c)$ resources will initially be consumed at a faster rate $(\partial q_0/\partial I^m > 0)$ since stocks are believed to be large. If as a result, fringe suppliers are induced to sell most of their stocks in early time periods, the cartel can assume a monopoly position later on when fringe supplies have been depleted. Note that the cartel is allowed to change sales without compensating consumers who might be harmed in the process. This would be possible if futures contracts for sales were perfectly binding. Of course in order to be convincing, the cartel must sell certain quantities $q^m(\bar{I}^m, I^c, t)$ at artifically deflated prices in return for the monopoly position it hopes to attain in the future. Obviously, this is just one of several strategies that the cartel might pursue.

3.2 Cheating by Misrepresenting Initial Inventories

This procedure (P1) describes the simplest option for reporting initial inventories. Clearly, the strategy in which the cartel reports its actual initial inventory and then follows the market equilibrium path thereafter is feasible under (P1) and the following proposition asserts that this is, in fact, the optimal policy for the cartel to choose, even for more complicated strategies as well. For simplicity in the remainder of the paper we shall confine our attention to the decreasing elasticity case. To make some of the mathematics more tractable we consider a special case of (DE)

$$\text{(DE')} \quad p(q) = a - q^{\alpha+1}; \; -1 < \alpha < \infty, \; a > 0$$

The (DE') demand specification is reasonably general as it allows for convex and concave demand schedules.

Proposition 2: Given the partial information structure (I2) and the demand specification (DE') the best strategy for the cartel is to announce its actual initial inventories and follow the equilibrium sales path.

Proposition 2 is quite general. It rules out the optimality of any strategy which involves a single or repeated misrepresentation of existing cartel reserves.

Proof

First we demonstrate that any cheating strategy the cartel pursues that satisfies procedure (P1) is dominated by telling the truth. All cheating strategies must involve a period of time in which a strategy of the type described by (P1) is used, and we are therefore able to rule out the optimality of all possible cheating schemes.

(a) At some time t_0 the cartel announces its current inventories as being $I^m(t_0)$. We arbitrarily set $t_0 = 0$. Assume I^m is the actual initial cartel stock. Construct

$$(3.1)\quad V(\overline{I}^m, I^c, \tau) = \int_0^{\tau} e^{-rt}[p_1(q_1^m, q_1^c, t)\, q^m]dt + e^{-r\tau} W^m(I^m - \int_0^{\tau} q_1^m(\overline{I}^m, I^c, t)dt, I^c - \int_0^{\tau} q_1^c(\overline{I}^m, I^c, t)dt$$

where q_1^m and q_1^c are the unique equilibrium cartel and competitive fringe sales and p_1 is the equilibrium price, all as functions of $\overline{I}^m$, I^c and t. The function V(• •), is the discounted profit to the cartel which announces initial inventories $\overline{I}^m$, follows the Nash equilibrium corresponding to $(\overline{I}^m, I^c)$ from time 0 to τ, then announces its actual inventories at time τ, which are $I^m - \int_0^{\tau} q_1^m(t)dt$, and receives the payoff, $W(\cdot,\cdot)$, which is the sum of discounted profits earned by the cartel for a Nash equilibrium corresponding to existing inventories $(I^m(T), I^c(T))$. (b) Differentiating $V(\overline{I}^m, I^c, T)$ with respect to τ, we obtain

$$(3.2) \quad \frac{\partial V(\bar{I}^m, I^c, \tau)}{\partial \tau} = \{P_1(\tau)q_1^m(\tau) - rW^m(I^m(\tau), I^c(\tau)) - \frac{\partial W^m}{\partial I^m} q_1^m(\tau)$$

$$- \frac{\partial W^m}{\partial I^c} q_1{}^c(\tau)\}e^{-r\tau}$$

We wish to show $\frac{\partial V}{\partial \tau} < 0$ for $\bar{I}\,^m(0) \neq I^m\,(0)$.

For $\bar{I}\,^m(0) = I\,^m(0)$ we have the identity

$$(3.3) \qquad V(I^m, I^c, \tau) = W^m(I^m(0), I^c(0)), \ (\forall T)$$

so that

$$(3.4) \qquad \frac{\partial V(I^m, I^c, \tau)}{\partial \tau} = 0, \ (\forall \tau)$$

In appendix A we show that $\frac{\partial W^m(\tau)}{\partial I^m(\tau)} = MR_2^m\,(\tau)$ and $\frac{W^m(\tau)}{I^c(\tau)} = MR_2^m(\tau) - P_2(\tau)$

where $P_2(\tau)$ and $MR_2^m(\tau)$ are the constant discounted price and cartel marginal revenue for the new Nash equilibrium corresponding to initial inventories $(I^m(\tau), I^c(\tau))$.

Substituting for these terms in equation (3.4) yields

$$(3.5) \quad rW(I^m(\tau), I^c(\tau)) = (P_2(\tau) - MR_2^m(\tau)(q_2(\tau))$$

Substituting for $rW(I^m(\tau), I^c(\tau)$ in (3.2) yields

$$(3.6) \quad \frac{\partial V(I^m, I^c, \tau)}{\partial \tau} = P_1(\tau)q_1^m(\tau) - (P_2(\tau) - MR_2^m(\tau))\, q_2(\tau) - MR_2^m(\tau)q_1^m(\tau)$$

$$-(MR_2^m(\tau) - P_2(\tau))\ (q_2(\tau) - q_2^m(\tau)\}\ e^{-rt}$$

$$= (P_1(\tau) - P_2(\tau))q_1^m(\tau) + (P_2(\tau) - MR_2^m(\tau))(q_1(\tau) - q_2(\tau)\}e^{-rt}$$

From equation (3.6) we can verify that $\frac{\partial V(I^m, I^c, \tau)}{\partial \tau} = 0$ by noting that

$P_1(\tau) = P_2(\tau)$ and $q_1(\tau) = q_2(\tau)$ (equilibrium price and quantity paths do not change) when $\bar{I}^m(0) = I^m(0)$.

(c) We now wish to show

$$(3.7) \quad \frac{\partial V(\bar{I}^m, I^c, \tau)}{\partial \tau} < 0 \quad \text{for } \bar{I}^m \neq I^m$$

Suppose the demand schedule is linear with $\alpha = 0$ and $dp/dq = -1$. The second expression on the right hand side of (3.6) can be rewritten as

$$(3.8) \quad (P_2(\tau) - MR_2^m(\tau))(q_1(\tau) - q_2(\tau))$$

$$= (-\frac{dp}{dq}\, q_2^m(\tau))\ (q_1(P_1(\tau)) - q_2(P_2(\tau)))$$

$$= -\ (P_1(\tau) - P_2\ (\tau))\ (q_2^m(\tau))$$

which when combined with (3.6) yields

$$(3.9) \quad \frac{\partial V}{\partial \tau} = (P_1(\tau) - P_2(\tau))(q_1^m(\tau) - q_2^m(\tau)) \qquad ;\ \alpha = 0$$

Similarly, one can establish the following inequalities in the case (DE') when $\alpha \neq 0$:

$$(3.10) \quad \frac{\partial V}{\partial \tau} < (P_1(\tau) - P_2(\tau))\ (q_1^m\ (\tau) - q_2^m\ (\tau)), \quad \alpha\ \varepsilon(0, \infty)$$

$$(3.11) \quad \frac{\partial V}{\partial \tau} < (P_2(\tau) - MR_2^m(\tau))(q_1(P_1(\tau)) - q_2(P_2(\tau))\left(\frac{q_2^m(\tau) - q_1^m(\tau)}{q_2^m(\tau)}\right)$$

$$\alpha \varepsilon\ (-1,0)$$

In Appendix B we establish

$$(3.12) \quad q_1^m(\tau) \gtreqless q_2^m(\tau) \quad \text{as } \bar{I}^m \gtreqless I^m\ (0)$$

which together with the condition $\frac{\partial q(I^m, I^c, 0)}{\partial I^m} > 0$ is sufficient to establish

(3.13) $\frac{\partial V}{\partial \tau} < 0$ whenever $\overline{I}^{\,m} \neq I^m$

(c) Thus, telling the truth dominates any strategy involving an initial period of cheating by stock misrepresentation. This result generalizes to more complicated strategies. Suppose the cartel announces different inventories at N various times, assuming fringe suppliers naively continue to adjust sales in accord with these inventories. From equation (3.13), working backwards, the best policy for the cartel is to reveal its true inventory in the last period N as well as period N-1. But the same reasoning which led to equation (3.13) implies that the cartel should reveal its true stock level in period N-2, given that actual inventories are to be announced for the periods N-1 and N. Repeating the argument for periods N-3, N-4...and so on, we can show that telling the truth dominates any cheating strategy, and Proposition 2 is established.

When cartel sales can be monitored, cheating doesn't work because the cartel must invest too much to convince fringe suppliers that its announced inventories are valid. For example, announcing very large inventories $\overline{I}^{\,m}$ in the hopes of increasing initial sales and inducing fringe suppliers to sell out early, requires that the cartel sell substantial quantities at artificially deflated prices. The loss incurred by the sale of these quantities outweights the gains from a future monopoly position for the cartel. Stated in another way the response of fringe sales to different inventory levels is too sluggish for cheating to work. For example, Salant (1976, pg. 1085) shows that if demand is linear, then $q_1^c(t)$ is independent of the cartel's actual or announced reserve holdings. Thus, the cartel is unable to change fringe production at all by misrepresenting initial reserves. In general, continuous monitoring of output makes lies about reserves unprofitable and thus not worth worrying about. In Lewis and Schmalensee (1978) we assume that initial reserves are known, but that cartel output is unobservable. There, we find it is profitable for the cartel to lie either by over-stating or under-stating its sales for a finite early period.

4. NONCOOPERATIVE OLIGOPOLY

4.1 Assumptions and Notation

In this section, we briefly consider some properties of noncooperative oligopoly models of renewable resource markets. Aside form some analysis by Salant (1976, Appendix B), this appears to be the first reasonably systematic examination of such models.

Assume there exist N "firms" (which may be ordinary firms, nations, partial cartels, or other reserve owners) capable of producing the resource. Let total initial reserves be I, let firm i's initial reserves be I_i, and let $S_i = I_i/I$, for all i. Similarly, let $q_i(t)$ be firm i's rate of production at time t, let Q(t) be the sum of the $q_i(t)$ and let $\sigma_i(t) = q_i(t)/Q(t)$. We assume that firm i's unit extraction cost is c_i, a constant. As before, $P(t) = P[Q(t)]$ is the market price and r is the rate of discount.

Each firm is assumed to choose the time path of its output so as to maximize the present value of its net revenue, taking as given the outputs of the other firms. In equilibrium, no single firm can profitably alter its output strategy. It is clear that for firms, the discounted net marginal revenue at all t such that $q_i(t) > 0$ must be equal, and this common value must at least equal the discounted net price at all other times. If not, since the outputs of all other firms are taken as given, a shift in production policy would raise the present value of net revenue. If we let $a[P(t)] = -dP/dQ$, the equilibrium

(4.1a) $$P(t) - q_i(t)\ a[P(t)] - c_i = \lambda_i e^{rt}, \quad \text{if } q_i(t) > 0,$$

(4.1b) $$P(t) - c_i \leq \lambda_i\ e^{rt}, \quad \text{if } q_i(t) = 0, \text{ and}$$

(4.2) $$I_i = \int_0^\infty q_i(t)\ dt,$$

for $i = 1,\ldots,N$, along with the demand constraint $P(t) = P[Q(t)]$. It is clear that λ_i measures the marginal (private) value to firm i of additional reserves at time zero.

4.2 Zero Extraction Cost

In order to focus on the implications of various elements of the general situation defined above, it will be useful to begin by assumimg that all the c_i are zero. Extension to the case of equal positive C_i is not difficult. The simplest situation is then the case of constant elasticity of demand, case CE. If ε is the absolute value of demand elasticity, conditions (4.1a) may be written as

(4.3) $$P[1 - \sigma_i/\varepsilon] = \lambda_i e^{rt}.$$

At some time t, let $\bar{\lambda}$ be the simple average of the λ_i of those firms producing at a positive rate. Differentiation of (4.3), summation across i, and division then yields

(4.4) $$\dot{P}/P = r = r(\bar{\lambda}/\lambda_i)\,[1 - \sigma_i/\varepsilon]/[1 - 1/N\varepsilon].$$

From this it is immediate that σ_i must be constant for all i in equilibrium, so that $\sigma_i = S_i$ for all i and t. It is clear from (4.3) that for such an equilibrium to exist, ε must exceed the largest of the S_i. All firms then produce forever, and price rises at the rate of interest. In this case, as was noted in Section 2 above, the function P(t) is independent of market structure.

In fact, explicit solution is easily obtained in this special case. If we choose units so that the demand curve can be written as $Q = P^{-\varepsilon}$, we obtain

(4.5a) $$P(t) = (r\,\varepsilon I)^{-1/\varepsilon} e^{rt},$$

(4.5b) $$\lambda_i = (r\varepsilon I)^{-1/\varepsilon}[1 - S_i/\varepsilon],$$

(4.5c) $$V_i = I_i(r\varepsilon I)^{-1/\varepsilon},$$

where V_i is the present value of firm i's receipts. It is easy to show that $\lambda_i = \partial V_i/\partial I_i$, holding constant the reserve levels of other firms. Note that smaller firms (as measured by reserves) have larger λ_i and thus greater incentives for exploration. Thus, even though the price path overtime in this model is independent of the distribution of reserves among producers, it seems likely that lower concentration would lead to more exploration in an expanded model.

Now let us consider case DE, in which demand elasticity falls with total output. The analysis is simplified by assuming $S_i = 1/N$ for all i. Let P(0) = F, the choke price, be finite as before. By the symmetry of the problem, we can look for an equilibrium in which $\sigma_i = 1/N$ for all i and t. If T is the total time taken to exhaust the resource, the equilibrium conditions can be written as

$$(4.6)\qquad Fe^{-ru} = P - (1/N)Q(P)a(P) = mr(P), \quad 0 \leq u \leq T$$

$$(4.7)\qquad \int_0^T Q[P(t)]\,dt = I.$$

The assumption that demand elasticity rises with P can be seen to imply

$$(4.8)\qquad a'Q < (\varepsilon+1)/\varepsilon$$

To establish the existence of a noncooperative equilibrium, we follow Salant's (1976) method of proof. Assume there exists a price, $\hat{P}$, at which $N\varepsilon = 1$. For prices below $\hat{P}$, mr is negative, so that $P > \hat{P}$ at all times. Differentiation of (4.6) then establishes

$$(4.9)\qquad \dot{P} = rP\,\frac{N\varepsilon - 1}{\varepsilon[N+1) - a'Q]}.$$

From (4.8), P is increasing for $P > \hat{P}$, so that Q is falling at all times. Further, (4.8) and (4.9) establish that $(\dot{P}/P) < r$ at all times and that in the limit as N increases, $(\dot{P}/P)$ approaches r.

Suppose one picks a $\tilde{P}_0$ and allows P to grow according to (4.9) until it equals F. As long as demand is smooth, the total production along this path, $\tilde{I}$, will be a continuous function of $\tilde{P}_0$. Clearly $\tilde{I}(F) = 0$. As $\tilde{P}_0$ approaches $\hat{P}$ from above, I must increase without bound, along with the total time the system spends with price arbitrarily near $\hat{P}$. It is easily seen that $\tilde{I}$ is monotone decreasing in $\tilde{P}_o$. Thus there must exist a P_0 such that $\tilde{I}(P_0) = I$, and we have established the first part of

<u>Proposition 3</u> If $c_i = 0$ and $S_i = 1/N$, for $i = 1,\ldots,N$, if demand is sufficiently smooth, and if there exists $\hat{P}$ such that $N\varepsilon(\hat{P}) = 1$ then there exists a unique noncooperative oligopoly equilibrium satisfying (4.6) - (4.7) in case DE. Price increases at less than the rate of interest. Increases in N(a) lower T, (b) raise P_0, (c) lower total profits, (d) raise the marginal value of reserves to each firm, λ, and (e) raise the discounted value of net welfare.

Let us first prove (a) and (b). We want to show that if $N' > N$, then $T' < T$ and $P_0' < P_0$, where primes are used to denote quantities associated with N', and unprimed quantities are associated with N. That is, increasing the number of firms with reserves held fixed leads to more rapid extraction. For $P > \hat{P}$, mr is easily seen to be increasing in P under DE. For fixed positive u (time until exhaustion), (4.6) then implies that $P' < P$, so that $Q' > Q$. Given the total output constraint, (4.7), this implies that T' must be less than T. Since P is a monotone function of time, (4.7) can be transformed via a change of variable to

$$(4.7') \qquad \int_{P_0}^{F} [Q(P)/\dot{P}(P)]\, dP = I.$$

Suppose $P'_0 = P_0$. From (4.8) and (4.9), $\dot{P}'(P) > \dot{P}(P)$ for $P\varepsilon[P_0,F)$, and $Q(P) = Q'(P)$. Thus less total output is produced with N' than with N. The only way to satisfy the constraint is to have $P_0' < P_0$, and (a) and (b) of Proposition 3 are established.

Part (d) of Proposition 3 follows immediately from (a) and (4.1a). With more competition, the incremental value of reserves rises for each firm. Heuristically, there are two opposed forces here: price tends to be lower, but marginal revenue tends to be closer to price. The second of these dominates. Exploration incentives are enhanced with other firms, even though prices are lower.

We can most readily establish (c) and (e) of Proposition 3, by writing the equilibrium output path as Q(N,t) for fixed I. We suppose that demand is smooth enough that Q is differentiable in N. Then (4.6) can be re-written as

$$(4.6') \qquad P[Q(N,t)] - (1/N)Q(N,t)\, a\{P[Q(N,t)]\} = \lambda e^{rt}.$$

Differentiation with respect to N for fixed t and substitution for aQ/N^2 yield

$$(4.10) \qquad Q_N(N,t)aN^2[N+1-a'Q] = P - [N(\partial\lambda/\partial N) - \lambda]e^{rt}.$$

By (4.8) and $P > \hat{P}$, the quantity multiplying Q_N is positive. From (b) of Proposition 3, $Q_N(N,0) > 0$, so that the right-hand side of (4.10) is initially positive. But since P grows at a rate less than r, (4.10) then implies that Q_N is initially positive, reaches zero in finite time, and is negative thereafter.

Total discounted profit for all firms can be written as

$$(4.11) \qquad V = \int_0^T P[Q(N,t)]Q(N,t)e^{-rt}\, dt.$$

Since Q(N,T) = 0 for any N, and P(0) = F is finite, we have

$$(4.15) \quad \partial V/\partial N = \int_0^T [P-aQ]Q_N(N,t)e^{-rt}dt = N\int_0^T \lambda Q_N(N,t)dt-(N-1)\int_0^T (Pe^{-rt})Q_N(N,t)dt.$$

using (4.6'). The first integral is zero from (4.7) and Q(N,T) = 0. Since P is always increasing at a rate less than r, (Pe^{-rt}) is everywhere decreasing. Since Q_N is positive for small t and negative for large t, integrating to zero overall, it follows that the second integral on the right of (4.15) is positive. Thus $\partial V/\partial N$ is negative, as was to be shown. This is an intuitive result; more competition lowers profits.

Finally, the sum of total discounted consumers' surplus and firm's rents is given by

$$(4.16) \qquad W = \int_0^T \left\{ \int_0^{Q(N,t)} P(x)\, dx \right\} e^{-rt}\, dt.$$

Since $Q(N,T) = 0$, differentiation yields

$$(4.17) \qquad \partial W/\partial N = \int_0^P (Pe^{-rt}) Q_N(N,t)\, dt,$$

which is positive on the reasoning above. We have thus completed the proof of Proposition 3 and shown that under our assumption, increased competition improves performance. The asymmetric case where initial reserves and unit extraction costs are not equal is considerably more complicated and is being investigated in a subsequent paper by the authors.[9]

FOOTNOTES

1. See, for instance, Scherer (1970, ch. 5). It is fair to say that these are the only widely used formal models of intermediate structures except for those concerned with entry, cartel stability, or differentiated products.

2. Interesting analyses of oligopoly pricing of renewable resources appears in Clark (1978) and Levhari and Mirman (1978). Stiglitz (1976) has analyzed the relation between competitive and monopoly behavior in nonrenewable resource markets.

3. In two interesting and important recent papers, Pindyck (1977, 1978) computes the wealth-maximizing price policies for a number of actual incomplete cartels, taking into account distributed lags in demand. In order to obtain tractable models, however, Pindyck assumes that competitors of these cartels behave myopically. In both Salant's (1976) analysis and the models that follow, all producers are fully rational, but demand curves are stationary.

4. For work on these and related aspects of resource markets, see Eckbo (1976), Hnyilicza and Pindyck (1976), Kosobud and Stokes (1977), Pakravan (1976), Pindyck (1977, 1978), and Schmalensee (1976).

5. For specifications (CE) and (IE) choke prices are infinite so that we need to assume $\lim_{q \to 0} p(q)q < \infty$.

6. These are equivalent to equations (1) - (4) in Salant (1976). Henceforth, time arguments are deleted where no confusion exists.

7. Clearly for the extreme case where $I^m = I$, the entire stock is allocated in a monopoly market and $\dot{p}/p > r$. If we change the distribution of initial reserves by decreasing I^m and increasing I^c we eventually encounter a situation in which all reserves are allocated under competitive conditions with $\dot{p}/p = r$. Recall from Lemma 2 that $\tilde{q}^c(0)$ is the initial sales rate which occurs when all resources are sold competitively with $\dot{p}/p = r$. When $(q^c, q^m > 0)$ we have $q^m = (1-K)\varepsilon(q)q$ by equation (2.8). Thus the largest value of I^m consistant with the competitive sale of all reserves is

$$\bar{I}^m = \int_0^\infty (1-K)\varepsilon(q)q\,dq$$

where

$$q(0) = \tilde{q}^c(0)$$

$$K = 1 - \frac{1}{\varepsilon(\tilde{q}^c(0))}$$

For $I^m > \bar{I}^m$, an initial period exists in which $q^c = 0$, $q^m > 0$.

8. Actually, it is sufficient for the cartel to just announce a quantity path, since its total initial reserves can be inferred from the sales path.

9. In the case $c_i = 0$, $i = 1,2,,,N$, and $S_i < S_j$ for some pair of firms we find that if a noncooperative equilibrium exists then (a) prices rise at less than the rate of interest, (b) firm i exhausts reserves and ceases production as q_1 falls to zero before firm j and (c) the initial price P_0 is higher the more unequal the initial distribution of reserves. If in addition, $0 < c_i < c_j$ for some i and j, then

the total discounted cost of producing the equilibrium total output path, $Q(t)$ is not minimized.

APPENDIX A

Comparative static derivations for the first five rows of Table 1 are signed following a method developed by Hartwick (1977) for the (DE) demand specification, noting that the other cases (CE) and (IE) are handled in a similar manner. Details of the other derivations in Table 1 are available upon request from the authors.

Let us first consider the first two lines in Table 1. From equations (2.8), (2.10), and (2.16) we have

$$I^c = \int_{q_1}^{q_0} - [1 - (1 - K(q_1)\ \varepsilon(q)]q/\dot{\tilde{q}}^c dq \tag{A.1}$$

$$I^m = \int_{q_1}^{q_0} - [1 - k(q_1))\varepsilon(q)]q/\dot{\tilde{q}}^c dq + \int_{0}^{q_1} - q/\dot{\tilde{q}}^m dq \tag{A.2}$$

Total differentiation of equations (A1) and (A2) yields a set of equations which can be represented in matrix form as

$$\begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{bmatrix} dq_0 \\ dq_1 \end{bmatrix} = \begin{bmatrix} dI^c \\ dI^m \end{bmatrix} \tag{A.3}$$

where

$$\alpha_{11} = -(1-(1-k(q_0))\varepsilon(q_0)q_0/\dot{\tilde{q}}^c(q_0) \quad > \quad 0$$

$$\alpha_{12} = -\int_{q_1}^{q_0} \varepsilon(q)q(\frac{dk}{dq_1})/\dot{\tilde{q}}^c dq \quad < \quad 0$$

$$\alpha_{21} = -(1-k(q_1))\varepsilon(q_1)q_1/\dot{\tilde{q}}^c(q_1) \quad > \quad 0$$

$$\alpha_{22} = \int_{q_1}^{q_0} \frac{dk}{dq_1}\varepsilon(q)/\dot{\tilde{q}}^c dq - q_1/\dot{\tilde{q}}^m(q_1) \quad > \quad 0$$

$$\frac{dk}{dq_1} = \frac{\varepsilon'(q_1)}{\varepsilon^2(q_1)} < 0$$

Repeated application of Cramer's Rule yields,

(A.4) $\quad dq_0/dI^c \quad > \quad 0$

(A.5) $\quad dq_0/dI^m \quad > \quad 0$

(A.6) $\quad dq_1/dI^c \quad > \quad 0$

(A.7) $\quad dq_1/dI^m \quad > \quad 0$

(A.8) $\quad dq_0/dS = dq_0/dI^m - dq_0/dI^c \quad < \quad 0$

(A.9) $\quad dq_1/ds = dq_1/dI^m - dq_1/dI^c \quad > \quad 0$

Where a comparison of relative magnitudes in equations in (A4) and (A5), and (A6) and (A7) are required to sign dq_0/dS and dq_1/dS. Equations (A4), (A5), (A8) and the condition $p' < 0$ imply the signs appearing in the second row of Table 1. From equations (2.1) and (2.3) we have that whenever $q^c(t)$, $q^m(t) > 0$

(A.10) $$MR_0^m = k\ p_0$$

Differentiating (A10) first with respect to I^m and then I^c yields

(A.11) $$\partial MR_0^m/\partial I^m = (dk/dq_1)\ (\partial q_1/\partial I^m)p_0 + k\partial p_0/\partial I^m < 0$$

(A.12) $$\partial MR_0^m/\partial I^c = (dk/dq_1)\ (\partial q_1/\partial I^c)p_0 + k\partial p_0/\partial I^c < 0$$

(A.13) $$\partial MR^m/\partial S = \partial MR^m/\partial I^m - \partial MR^m/\partial I^c \qquad < 0$$

where a comparison of relative magnitudes in (A11) and (A12) is required to sign (A13).

A similar approach is used for lines 3-5 in Table 1. The functions $W^m(I^m, I^c)$ and $W^c(I^m, I^c)$ are defined by

(A.14) $$W^m = \int_0^{T_2} e^{-rt}\ p(q^e+q^m)q^m dt$$

(A.15) $$W^c = \int_0^{T_1} e^{-rt}\ p(q^c+q^m)q^c dt$$

where T_1 and T_2 are the time horizons over which fringe suppliers and the cartel operate respectively, with $T_2 > T_1$.

Straightforward differentiation of W^m with respect to I^m yields:

(A.16) $$\partial W^m/\partial I^m = \int_0^{T_2} e^{-rt}[p\ dq^m/dI^m + dp/dq\ (dq^m/dI^m + dq^c/dI^m)q^m]dt$$

$$+ \frac{\partial T_2}{\partial I^m} \quad e^{-rT_2}\ p(T_2)q^m(T_2)$$

The second term on the righthand side of (A16) is zero since $q^m(T_2) = 0$. Rearranging (A16) we obtain

$$\partial W^m/\partial I^m = \int^{T_2} e^{-rt}[MR^m dq^m/dI^m + (MR^m - p)dq^c/dI^m]dt$$

In order to preserve our inventory constraints we have from equation (2.2)

(A.18) $$\frac{\partial I^c}{\partial I^m} = 0 = \int_0^{T_1} \frac{\partial q^c}{\partial I^m} dt + \partial T_1/\partial I^m q^m(T_1)$$

(A.19) $$\partial I^m/\partial I^m = 1 - \int_0^{T_2} \frac{\partial q^m}{\partial I^m} dt$$

Note that $q^m(T_1) = 0$ in order to satisfy equations (2.1) and (2.3).

Using equations (2.11), (2.3), (A18) and (A19) we obtain

(A.20) $$\partial W^m/\partial I^m = MR_0^m \int_0^{T_2} dq^m/dI^m dt + (MR_0^m - p_0) \int^{T_1} dq^c/\partial I^m dt = MR_0^m > 0$$

Straightforward but somewhat tedious application of this same method can be shown to yield the following derivatives with the indicated signs

(A.21) $$\partial W^m/\partial I^c = MR_0^m - p_0 < 0$$

(A.22) $$\partial W^m/\partial S = \partial W^m/\partial I^m - \partial W^m/\partial I^c = p^0 > 0$$

(A.23 $$\partial W^c/\partial I^m = \int_0^{T_1} e^{-rt}[dp/dq \cdot q^c \cdot dq/dI^m]dt < 0$$

(A.24) $$\partial W^c/\partial I^c = p_0 + \int_0^{T_1} e^{-rt}[dp/dq \cdot q^c dg/dI^c]dt > 0$$

(A.25) $$\partial W^c/\partial S = \partial W^c/\partial I^m - \partial W^c/\partial I^c < 0$$

(A.26) $$\partial(W^m + W^c)/\partial S = \partial W^m/\partial S + \partial W^c/\partial S > 0$$

APPENDIX B

The condition

(B.1) $$q_1^m(T) \lesseqgtr q_2^m(T) \qquad \text{as } \bar{I}^{\,m} \lesseqgtr I^m(0)$$

for (DE) specification is derived as follows.

(a) Let $p(t,\bar{I}^{\,m}, I^c)$ and $MR^m(t;\bar{I}^{\,m}, I^c)$ be the Nash equilibrium price and marginal revenue paths. From Table 1, $\frac{\partial P(0)}{\partial \bar{I}^{\,m}} < 0$ implying an initial downward shift in prices as $\bar{I}^{\,m}$ increases. In addition, $MR_0^m(0) = e^{-rt}F$. where F is equal to the choke price and T is the time it takes to exhaust total inventories. Since $\frac{\partial MR^m(0)}{\partial \bar{I}^{\,m}} = -re^{-rt} F \frac{\partial T}{\partial \bar{I}^{\,m}} < 0$ We conclude that $\frac{\partial T}{\partial I^m} > 0$ which combined with our earlier argument can be shown to imply that $\frac{\partial p(t)}{\partial \bar{I}^{\,m}} < 0$, $(\forall t)$ or $\frac{\partial q(t)}{\partial \bar{I}^{\,m}} > 0$. Since $q^m(t) = q(t)$ for $t > t_1$ we have

(B.2) $$\frac{\partial q^m(t)}{\partial \bar{I}^{\,m}} > 0 \qquad t > t_1$$

(b) We now wish to sign $\frac{\partial q^m(t)}{\partial \bar{I}^{\,m}}$ for $t \geq t_1$ which we will do examining the sign of $\frac{\partial q^c(t)}{\partial \bar{I}^{\,m}}$. Since $\frac{\partial q(t)}{\partial \bar{I}^{\,m}} > 0$ we have

(B.3) $$\text{sign } \frac{\partial q^c(t)}{\partial \bar{I}^{\,m}} = \text{sign} \left(\frac{\partial q^c(t)}{\partial q(t)}\right)\left(\frac{\partial q(t)}{\partial \bar{I}^{\,m}}\right) = \text{sign } \frac{\partial q^c}{\partial \bar{q}(t)}$$

Given (DE) and (2.1) and (2.3) we have

(B.4) $$q^m(t) = \frac{p(t) - MR^m(t)}{(\alpha+1)\ q(t)^\alpha} = \frac{e^{-rt}(p(0) - MR^m(0))}{(\alpha+1)\ q(t)^\alpha}$$

Straightforward differentiation of $q^c(t)$ with respect to $q(t)$, substituting for $q^m(t)$ from (B.4) yields

$$\text{(B.5)} \qquad \frac{dq^c(t)}{dq(t)} = 1-e^{-rt}\left[\frac{\frac{d}{dq(0)}(p(0)-MR^m(0))e^{-rt}(\frac{q(t)}{q(0)})^{\alpha} - \frac{\alpha}{q(t)}(p(0)-MR^m(0))}{(\alpha+1)\ q(t)^{\alpha}}\right]$$

From (B.5) we can establish that

$$\text{(B.6)} \qquad \frac{d}{dq(0)}\ (P(0)-MR^m\ (0)) > 0$$

To see this suppose $\frac{d}{dq(0)}(p(0)-MR^m(0))<0$. Then if $\alpha \geq 0$, (B.5) implies $\frac{\partial q^c(t)}{\partial \bar{I}^{\,m}} > 0$ $(\forall t)$ which violates (2.3). Differentiating (B.5) with respect to time we obtain

$$\text{(B.7)} \qquad \frac{d}{dt}\left\{\frac{dq^e(t)}{dq(t)}\right\} =$$

$$r\left\{\frac{dq^e}{dq(t)} - 1\right\} - e^{-rt}\left\{\frac{-re^{-rt}\frac{d}{dq(0)}(P(0)-MP^m(0))(q(0))^{-\alpha} + \alpha(\alpha+1)q^{-(\alpha+2)}\frac{dq}{dt}(P(0)-MP^m(0))}{\alpha+1}\right\}$$

If $\alpha < 0$ and $\frac{dq^c(t)}{dq(t)} \leq 0$ for some t' then (B.7) implies $\frac{dq^c(t)}{dq(t)} < 0$ for all $t \geq t'$. Clearly $\frac{\partial q^c(t)}{\partial q(t)} \leq 0$ for some $t' \in (0,\ t_1)$ (otherwise (2.3) is violated) but this implies $\frac{\partial q^c(t_1)}{\partial q(t_1)} < 0$ which is not an allowable variation since $q^c(t_1) = 0$. Consequently (B.6) must hold.

Given (B.6) we can now establish the signs of $\frac{\partial q^c(t)}{\partial q(t)}$ and $\frac{\partial q^m(t)}{\partial q(t)}$. For $\alpha < 0$ it follows from (B.5) that $\frac{\partial q^c(t)}{\partial q(t)} < 1$ which implies $\frac{\partial q^m}{\partial q(t)} > 0$ for all t. For $\alpha \geq 0$ we also have that $\frac{\partial q^c(t)}{\partial q(t)} < 1$ for $t \epsilon [0,t_1]$, for if $\frac{\partial q^c(t)}{\partial q(t)} > 1$ at some $t' < t_1$, (B.7) implies $\frac{\partial q^m(t)}{\partial q(t)} < 0$

for all $t' < t \leq t_1$. But given (B.2) $q^m(t)$ is discontinuous at t_1 implying that $MR^m(t_1^-) > MR^m(t_1)$ which violates (2.3). Thus we have established

$$\text{(B.8)} \qquad \frac{\partial q^m(t)}{\partial q(t)} > 0, \ (\forall t)$$

Condition (B.1) now follows immediately from (B.8) and fact that $\frac{\partial q(t)}{\partial \bar{I}^m} > 0$.

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